We're Poppin' for Math

By Brenda Levert

Subject: Algebra, real-world connections

Audience: Teachers, teacher educators

Grade Level: 7–8 (Ages 12–13)

Technology: Graphing calculators, spreadsheets, word processing software

Standards: *NETS•S* 3–6; *NETS•T* II, III (http://www.iste.org/standards). *NCTM* Grades 6–8 Algebra, Data Analysis and Probability, Problem Solving, and Communication (http://standards.nctm.org/document).

Each year, students in my seventh- and eighth-grade math classes plan and organize a schoolwide popcorn sale. This activity brings to life mathematical concepts learned in the classroom.

By transferring textbook mathematics to a real-world situation, my students learn to value the mathematics being studied and are able to understand how it can affect them in the future. To conduct this lesson, we use several technologies, including the TI-83 Plus to find the best selling price for the popcorn and create charts and graphs to support decision making, and Microsoft Excel and Word to organize data.

Seeding the Activity

Throughout the year, I focus on helping students understand why we learn mathematics. I am always striving to answer the familiar question posed each year, "When am I ever going to use this?" If I am to teach my students to value and appreciate mathematics I must help them see its relevance in their lives. With this in mind and with the understanding that children's learning is built from prior learning, I developed the project We're Poppin' for Math as a culminating activity for my unit on Solving Systems of Linear Equations. The activity requires the students to transfer their knowledge of solving systems of linear equations to orchestrate a successful popcorn sale. We begin the unit after my eighth-grade algebra students have successfully completed the unit on analyzing linear equations. We spend a few days investigating and planning the popcorn sale. The actual sale takes place in one day.

To capture my students' attention, I begin the lesson by posing the question, "How many of you sold lemonade as a child?" Often many of them raise their hands and discuss their experiences. I then ask them about their profit on those sales. We discuss the

terms *cost* and *revenue*. The students admit they never worried about their cost because they would simply take what they needed from their pantries at home. We agree that although it did not cost *them* anything, it cost someone. They usually laugh and say, "Yeah, our parents."

Investigating the Problem

I begin by having students work in small groups to investigate a problem called Popcorn that I modified from the Dale Seymour Publications book *Graphing Power*. I stress that this activity serves as a model to help guide and direct them in their upcoming assignment to plan and execute a schoolwide popcorn sale. The modified problem reads:

> Your class decides to sell bags of popcorn as a fundraiser. The cost to rent the popcorn machine is $15, and it costs 10¢ a bag to produce the popcorn. The class wants to sell the popcorn for 25¢ per bag. Using this information, calculate the cost, revenue, and profits for 0, 1, 10, 20, and 130 bags of popcorn.

The students consider the problem and find the values using basic computations.

Then, the activity asks students to derive functions to represent the cost, revenue, and profit. Students answer questions that help them understand what each function means. For example, the activity asks students to enter the cost function into their graphing calculator and generate a graph. Then the activity asks, "What does the x-axis represent?" and "What does the y-axis represent?" They then enter the other functions into their calculators and explore the results. This helps students connect multiple representations of their data.

Students soon discover the break-even point, and we discuss what this means in relation to the project and sale of popcorn. I relay to the students that finding the best selling price is critical to having a successful business. Students begin to realize that if they sold the popcorn for 25¢ a bag, considering their cost, they would have to sell 100 bags to break even. I pose the question "If we break even after selling 100 bags, how many bags must we sell to realize a profit?" Students answer "101." Familiar responses are, "That is too much work for such a small profit," or, "We only have about 400 students in our whole school. There's no way we can sell that much popcorn."

We use the table on the graphing calculator to find our maximum profit. Students get excited and begin to interject their ideas. Someone always suggests, "We could raise the price." That suggestion leads to, "How much?" Students discuss what they feel would be the best price. I steer the discussion to help the students decide to survey the school. We also agree that we must find our cost for this project. How much does a one-pound bag of popcorn kernels cost? How much does a gallon of popcorn oil cost? Where can we get a popcorn machine? What other items are needed.

This is the point at which I get very excited about the lesson. It is at this point that I can tell they have successfully transferred textbook knowledge to a real-world situation.

Involving Others

We then call on the expertise of the other mathematics classes in the school. The seventh graders are eager to participate. We explain what we need and ask the seventh graders to survey the school, compile the data, and give us a report of their investigation. The seventh graders survey the school, asking the students if they would be willing to pay 50¢ for a bag and which type of popcorn they would prefer—butter, plain, caramel, or cheese.

Using Microsoft Excel, the seventh graders organize the data to report back to the eighth grade Algebra 1 students. This activity helps the seventh graders extend their understanding of analyzing data by creating charts and graphs. Also, the spreadsheet activity helps students work

Popcorn Sale 2003

Part I

Your class decides to sell bags of popcorn as a schoolwide fundraiser. The cost of the materials is $25.23. In addition, you will need to pay 2¢ for each bag used. Determine a selling price for a bag of popcorn. Justify your rationale for determining your selling price.

1. Write a function for determining cost. Cost = _____
2. Write a function for determining the revenue. Revenue = _____
3. Write a function for determining the profit. Profit = _____
4. Enter your cost and revenue functions at Y1 and Y2, respectively. Graph the functions simultaneously. What does the point of intersection tell you?
5. What is the least number of bags of popcorn you must sell to make a profit? How did you determine this value?
6. Enter your profit function as Y3. Graph the function. If your class wants to make $100 profit, how many bags of popcorn do you need to sell? How did you arrive at this value?
7. Can you have a negative profit? What does this mean?

Part II

Create a Word document to describe and analyze your problem. Include screens from your calculator to verify your findings. Use verbal, tabular, graphical, and symbolic representations to explain your data.

Students use a real problem to guide their investigation of linear equations.

fluently with decimals, fractions, and percents. The seventh graders report the information to the Algebra 1 class, and this starts the ball rolling. Interestingly enough, the seventh graders usually uncover the same findings year to year—buttered popcorn is the most popular.

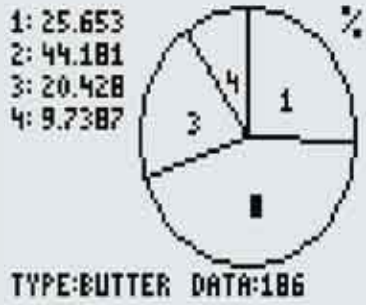
Determining the Costs

Once students collect and analyze all the data, they are ready to orchestrate their sale. They feel confident in their knowledge of solving systems of linear equations and can connect their knowledge and studies to a real-life situation. This type of activity motivates the students to learn the concepts being taught. Every year, students' eagerness to participate is evidenced by their comments and questions.

The last time I did this activity, one topic of discussion was centered on lowering the cost. A student brought in price quotes from a local movie theater and shared the prices of their popcorn. She also had a sample of the popcorn bag along with the price per bag. Another student said she would try to get her mother's employer to donate supplies. These ideas, along with other strategic thinking, proved that students knew if they could lower their cost, they could ensure a higher profit.

The class worked in small groups to collect and analyze data to determine the cost. We decided to purchase a large bag of kernels, 600 paper bags, and two gallons of butter-flavored popcorn oil and to use the school's Parent Teacher Association popcorn machine. (The school donated the butter-flavored salt.)

We calculated our initial cost (for kernels and oil) as $25.23 and used it as the y-intercept in the slope intercept form, y = mx + b, because these goods were perishable. We decided any bags left over could be used for next year, so we represented this in-



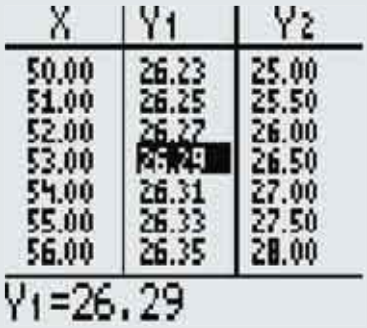
My class is having a popcorn sale. We went to each class and asked the students, out of cheese, butter, caramel, or regular, which they liked the best. The most popular flavor was butter (2) 44.2%, followed by cheese (1) 25.7%, then caramel (3) 20.4%, and then plain (4) 9.7%.

—*Josh*

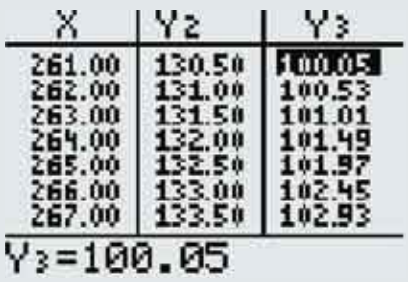
The seventh graders present their survey data to the eighth graders.

During this project we have decided that our selling price should be 50¢. Together with the cost of our materials and the added fee of 2¢ per bag, we have come to the conclusion that, we will need to sell about 53 bags to be at a breaking point. To reach a profit of $100, we will need to sell about 261 bags of popcorn.

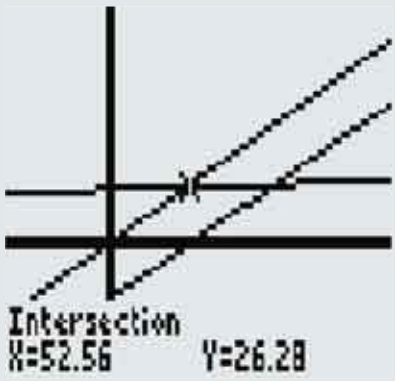
During this project we have decided that algebra is going to be a useful skill to know in the business world. We enjoyed figuring out the wonderful uses of algebra and hope to do more of these projects in the future. This was so much fun! Now I can't wait to sell the popcorn that we worked so hard to figure out the price for. I definitely recommend doing this with your class in the future.



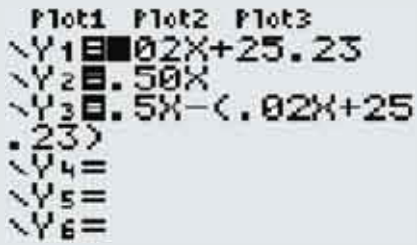
This is a picture of our table for Cost (Y1) and our Revenue (Y2); X represents the number of bags sold.



This is a continuation of the table. The Y3 represents the Profit. If we sell 261 bags we will make $100.05!



This is a graph of the information located on the table. Each line represents the equations in Y1, Y2, and Y3. This is the BREAK EVEN POINT!



Here are our linear equations. We derived these equations by placing our information in algebraic form.

By Michael, Heather, Will, and Renee

Eighth graders use the data gathered by the seventh graders and their technology tools to assess a price for the bags of popcorn. This report addresses part II on the assignment worksheet.

formation as our rate of change, the slope of the line, or *m*. We calculated each bag would cost 2¢ (or .02 dollars). The students derived the cost equation y = .02x + 25.23. We used the graphing calculator to explore various selling prices to investigate and determine the best selling price. The graphing calculator produced graphical and tabular representations and performed complex calculations so the students could focus on using functions to model patterns of quantitative change. For example, when the students used 25¢ for their selling price and viewed the table and graphs of their functions, they quickly realized they would need to sell a tremendous amount of popcorn to break even, let alone to see a profit. Students were able to avoid performing unnecessary calculations and painlessly substitute other values for the cost.

In this situation, technology proved to be a powerful tool to develop problem-solving skills by allowing students the ability to explore and investigate solutions. Students derived several functions to represent revenue and continued their investigation on their graphing calculators. By graphing the cost and revenue functions simultaneously, we explored various break-even points. Students used the table function to show the data in another form. Through their investigation, students decided 50¢ per bag of popcorn was the most reasonable price.

The students wrote the cost function y = .50x. They used the table to discover they would make $100 if they sold 261 bags of popcorn. This investigation helped them justify their conclusion that a popcorn sale would be profitable. They used screenshots from their TI-83 Plus to create a Word document summarizing their decision. They were instructed to include in their document graphical, tabular, and symbolic representations to explain their data. As stated in the NCTM's *Principles and Standards For School Mathematics*, using various forms of representation is "central to the study of mathematics. Students can develop and deepen their understanding of mathematical concepts and relations as they create, compare, and use various representations."

Using TI-Connect software and graph link cables, students transferred information from their graphing calculators to the computer. When the students' reports reflected that they had thoroughly investigated the problem and had valid information to support their decision, we set a date for the popcorn sale.

Selling the Popcorn

We sent a letter to the parents two weeks before the sale, explaining the project to solicit preorders.

We took preorders, and students recorded them according to the purchasers' lunch schedules. A week before the sale, I sent a reminder note home, and we collected more money and recorded more orders.

On the day of the sale, the students set up and popped popcorn. We filled preorders and boxed them in paper boxes according to their lunch schedule. We wrote the teacher's names on the box and waited for them to come into the cafeteria for lunch. In most cases, all the students in the class purchased a bag of popcorn. If not all students in a class had paid, we just included a couple of extra bags. We really did not want to leave anyone out. Our presale was such a success we did not compromise our profits by giving away bags of popcorn to these students. We noticed that if a couple of students had not prepaid, their teacher or a parent sent in extra money to cover this cost.

Extending the Learning

When all was done, students reported a profit of $108. Their predictions were correct. Students demonstrated their ability to solve systems of equations using graphical, tabular, and symbolic representations. Most important, I captured the interest of my middle school students. I believe this activity will help my students appreciate and value learning mathematics. Also, it helps them recognize technology as a valuable tool. Students took ownership in this project and expressed this was a fun and innovative way to "do math."

This activity's success can be attributed to the technological tools that generated visual images of mathematical concepts, facilitated organizing and analyzing data, and computed efficiently and accurately. As stated in NCTM's *Principles and Standards for School Mathematics*, "with calculators and computers, students can examine more examples or representational forms than are feasible by hand, so they can make and explore conjectures easily."

Also, the use of technology engaged the students by providing feedback necessary to create and validate decisions. This empowered the students and gave them confidence in their mathematical abilities.

Finally, this project encouraged students to go beyond the textbook applications and extend their learning to real-world applications through the use of technology.

Brenda Levert teaches Grade 7 and 8 math at the Academy for Academics and Arts in Huntsville, Alabama. She was a recipient of the 2000 Toyota TIME Grants for Teachers to purchase technology for her classroom. She is also a National Board Certified Teacher in early adolescence math.

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